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SOLUTIONS OF EXERCISES.

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About the vertices of an equilateral triangle three spheres are drawn with radii equal to the side of the triangle. Find the volume common to them all.

[*W. M. Thornton.*]

SOLUTION.

Let ABC be the triangle of the centres, O its ortho-centre, and N one of the apices of the solid. Draw BO to meet the surface in Q . The planes NOA , NOQ , AOQ , and the sphere-surface ANQ bound one-twelfth of the solid. The area of the sphere-surface ANQ is the difference between the area of the zone-surface ACN , whose angle ACN is arc $\tan 2\sqrt{2}$, and that of the right spherical triangle QCN , whose angles are arc $\tan 2\sqrt{2}$ and arc $\cot 2\sqrt{2}$.

The excess of this triangle is

$$E = \text{arc } \tan 2\sqrt{2} + \text{arc } \cot \sqrt{2} - \frac{1}{2} \pi.$$

Its area is

$$J = ER^2,$$

R being the radius of the sphere. The area of the zone-piece is

$$Z = \frac{1}{2} R^2 \text{arc } \tan 2\sqrt{2}.$$

Hence the curved surface of the portion of the solid under consideration is

$$\begin{aligned} Z - J &= R^2 \left(\frac{1}{2} \pi - \text{arc } \cot \sqrt{2} - \frac{1}{2} \text{arc } \tan 2\sqrt{2} \right) \\ &= R^2 \left(\text{arc } \tan \sqrt{2} - \frac{1}{2} \text{arc } \tan 2\sqrt{2} \right). \end{aligned}$$

The volume of the spherical pyramid which has this surface for its base is

$$\frac{1}{3} R^3 \left(\text{arc } \tan \sqrt{2} - \frac{1}{2} \text{arc } \tan 2\sqrt{2} \right).$$

The volume of the cone-segment whose base is NOA and apex B is

$$\frac{1}{3} R^3 \left(\frac{1}{4} \text{arc } \tan 2\sqrt{2} - \frac{1}{24} \pi \right).$$

The difference between these volumes is one-twelfth the volume of the solid common to three spheres. Hence

$$V = R^3 \left(\frac{1}{12} \sqrt{2} + 4 \text{arc } \tan \sqrt{2} - 3 \text{arc } \tan 2\sqrt{2} \right).$$

[*W. H. Echols.*]

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If

$$J = \begin{vmatrix} a_1 & b_1 & c_1 & \dots & h_1 \\ a_2 & b_2 & c_2 & \dots & h_2 \\ a_3 & b_3 & c_3 & \dots & h_3 \\ \dots & \dots & \dots & \dots & \dots \\ a_n & b_n & c_n & \dots & h_n \end{vmatrix}, \text{ and } D = \begin{vmatrix} \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} & \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} & \dots & \begin{vmatrix} a_1 & h_1 \\ a_2 & h_2 \end{vmatrix} \\ \begin{vmatrix} a_1 & b_1 \\ a_3 & b_3 \end{vmatrix} & \begin{vmatrix} a_1 & c_1 \\ a_3 & c_3 \end{vmatrix} & \dots & \begin{vmatrix} a_1 & h_1 \\ a_3 & h_3 \end{vmatrix} \\ \dots & \dots & \dots & \dots \\ \begin{vmatrix} a_1 & b_1 \\ a_n & b_n \end{vmatrix} & \begin{vmatrix} a_1 & c_1 \\ a_n & c_n \end{vmatrix} & \dots & \begin{vmatrix} a_1 & h_1 \\ a_n & h_n \end{vmatrix} \end{vmatrix};$$

then will

$$D = a_1^{n-2} J. \quad [T. M. Blakesslee.]$$

SOLUTION.

This exercise is given on p. 77 of Muir's Theory of Determinants. It may be got from the result of section 53 of that work; or it may be derived by a process analogous to that of the section referred to, as follows: Multiply each column after the first by a_1 ; add to each element of the second column, thus multiplied, $-b_1$ times the corresponding element of the first column; to each element of the new third column $-c_1$ times the corresponding element of the first column; ... to each element of the new n th column $-h_1$ times the corresponding element of the first column; and we have

$$a_1^{n-1} J = \begin{vmatrix} a_1 & 0 & 0 & \dots & 0 \\ a_2 & -a_2 b_1 + a_1 b_2 & -a_2 c_1 + a_1 c_2 & \dots & -a_2 h_1 + a_1 h_2 \\ a_3 & -a_3 b_1 + a_1 b_3 & -a_3 c_1 + a_1 c_3 & \dots & -a_3 h_1 + a_1 h_3 \\ \dots & \dots & \dots & \dots & \dots \\ a_n & -a_n b_1 + a_1 b_n & -a_n c_1 + a_1 c_n & \dots & -a_n h_1 + a_1 h_n \end{vmatrix}$$

$$= a_1 \begin{vmatrix} \begin{vmatrix} a_1 & b_2 \\ a_1 & b_3 \end{vmatrix} & \begin{vmatrix} a_1 & c_2 \\ a_1 & c_3 \end{vmatrix} & \dots & \begin{vmatrix} a_1 & h_2 \\ a_1 & h_3 \end{vmatrix} \\ \dots & \dots & \dots & \dots \\ \begin{vmatrix} a_1 & b_n \\ a_1 & c_n \end{vmatrix} & \begin{vmatrix} a_1 & c_n \end{vmatrix} & \dots & \begin{vmatrix} a_1 & h_n \end{vmatrix} \end{vmatrix} = a_1 D;$$

$$\therefore a_1^{n-2} J = D.$$

[W. B. Richards.]

Also solved by L. G. Weld and the proposer.

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Show that the attraction of a finite mass on one of its points is finite.

[*A. Hall.*]

SOLUTION.

Take the origin at the point attracted, and let $k \, dx \, dy \, dz$ be the element of mass, V the potential for a particle of the body, and r the distance of the particle from the origin. We have

$$V = \iiint \frac{k \, dx \, dy \, dz}{r}.$$

Put

$$x = r \cos u, \quad y = r \sin u \cos \lambda, \quad z = r \sin u \sin \lambda;$$

then

$$dx \, dy \, dz = r^2 \sin u \, du \, d\lambda \, dr,$$

and

$$V = \iiint k r \sin u \, du \, d\lambda \, dr.$$

The limits of integration are $u = 0$, to $u = \pi$; $\lambda = 0$, to $\lambda = 2\pi$; and $r = 0$, to the limits of the body. For the component of the force in the axis of x we have

$$X = \frac{\partial V}{\partial x} = \iiint \frac{k x \, dx \, dy \, dz}{r^3} = \iiint k \cos u \sin u \, du \, d\lambda \, dr,$$

with similar values for Y and Z . These components are finite, and therefore the resultant is finite. See Gauss, *Allgemeine Lehrsätze*, etc.

[*A. Hall.*]

EXERCISES.

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A body at distance r from the sun is moving with velocity v . Prove that the major axis of the orbit described is parallel to the direction of motion if, and only if, the velocity is "circular velocity for the distance r ."

[*Ellery W. Davis.*]

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If a horizontal beam of length $2a$ is supported at each end, and has a load in the form of an isosceles triangle, base $2a$, height b , a unit's thickness throughout, and heaviness unity; show that the deflection of the beam due to this triangular load is $\frac{2a^4b}{15EI}$.

[*T. U. Taylor.*]